Clock Composition by Wiener filtering Illustrated on Two Atomic Clocks

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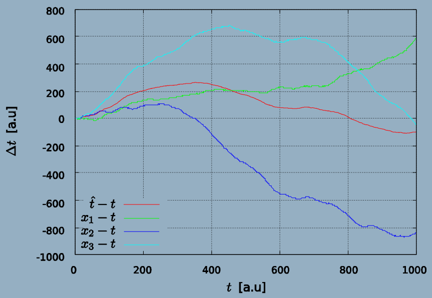
Clock ensembling

Introduction

- What is composite clock (ensembling)?



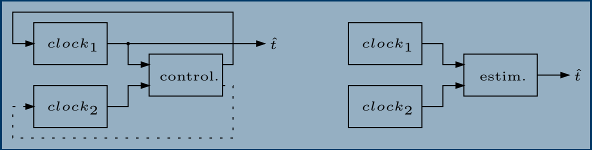
- compute “best” time given *N* noisy & drifting clock readings

Clock ensembling

Feedback vs. estimation

Two distinct approaches to clock ensembling:

- feedback control
  - corrected time is fed back into controller
  - e.g.: PLLs, FLLs,...
- estimation only
  - corrected time does not go back into the estimator



- due to separation principle:
  - $\sigma_{control} \geq \sigma_{estimation}$
  - $\Rightarrow$ estimation is better than control
    - (where applicable; e.g.: NTP, ACES)

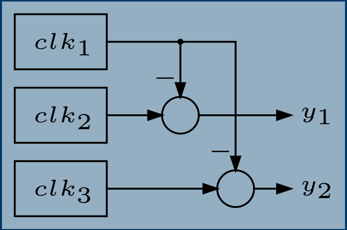
Clock model

- linear clock model assumed
  - $1/f$-noise is not linear (chaos or ∞ order) $\Rightarrow$ approximation
- equivalent descriptions
  - phase spectrum $\mathcal{S}_{xx}(f)$
  - state-space model $\mathbf{x}(t+1) = \Phi\mathbf{x}(t) + \mathbf{u}(t)$
  - transfer function $G(z)$
- MISO $\rightarrow$ SISO conversion by spectral factorization (spf($\cdot$))

Ensemble measurement

. . . and implied difficulty

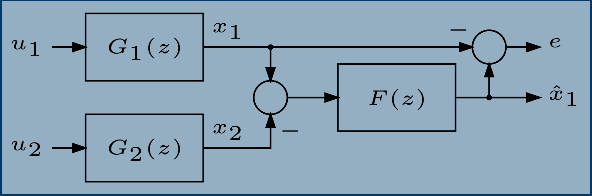
- only time differences can be measured
  - N clocks means $N - 1$ readings
  - measurement matrix is singular
  - system not completely observable



- non-observable system & all clocks drift ⇒ unbounded output error (ensemble drifts, too)

Linear estimator
Kalman & Wiener filters

- MSE optimal for linear system
- Kalman filter (KF) – can handle time-varying process
- Wiener filter (WF) $\equiv$ to KF for time-invariant
  - especially simple & insightful in SISO case ($N = 2$ clocks)



- $G_1 \leftrightarrow G_2 \Rightarrow F'(z) = 1 - F(z)$
- measurement noise may be incorporated into G_2

Wiener filter

- 3 variants
  - non-causal $F_{nc}(z) = \frac{S_{xy}}{S_{yy}} = \frac{B_1^* B_1}{C^* C}$
  - causal $F(z) = [S_{xy} W^*]_+ W = \left[\frac{B_1^* B_1}{AC^*}\right]_+ \frac{A}{C}$
  - finite-lag $F_T(z) = z^T [z^{-T} S_{xy} W^*]_+ W$
- design $=$ 2 operations
  - $C = \mathrm{spf}\,(B_1^* B_1 + B_2^* B_2)$ (root finding)
  - $[\cdot]_+$ (system of linear equations)

Design procedure
Clock-specific problems

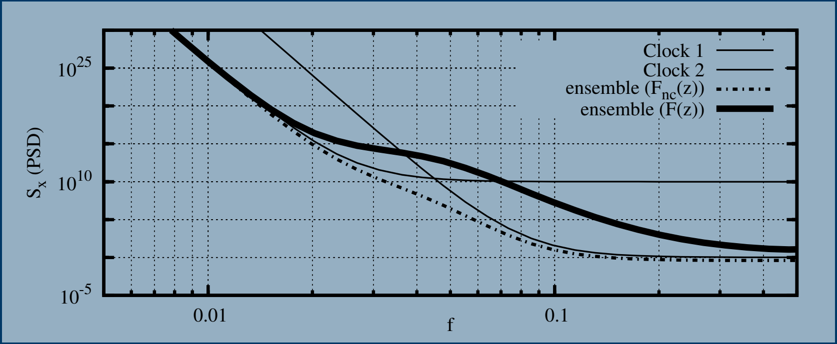
- marginally stable factors $A(z) = (z-1)^m \tilde{A}(z)$
  - treat as $A(z) = (z-(1-\epsilon))^m \tilde{A}(z)$
  - ϵ is only a notion to help splitting causal/non-causal
- huge frequency range
  - hard to perform spf($\cdot$)
  - solution: root-finding in arbitrary precision math

Non-causal WF

- best MSE of all
- needs to know future $y(t+1)\dots y(\infty)$
- $\equiv$ average weighted by $1/S_{11}, 1/S_{22}$
- $G_1 \propto G_2 \Rightarrow$ static weighted average

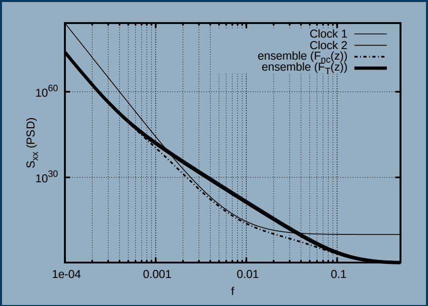
Causal WF

Example #1 – causal vs. non-causal



Causal WF

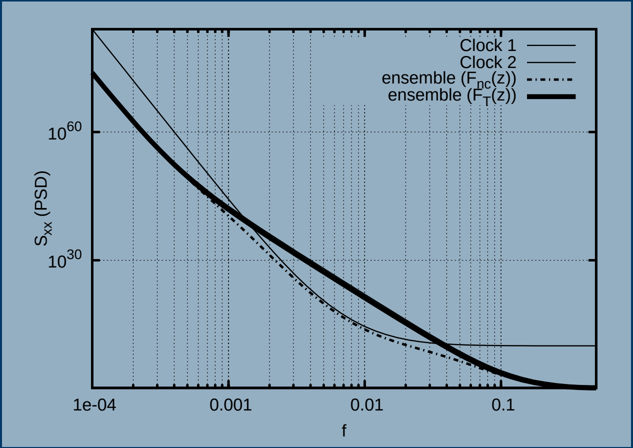
Example #2 – causal vs. non-causal



- $F(z)$ almost completely discards $x_2(t) \Rightarrow$ ensemble almost reduced to $clock_1$

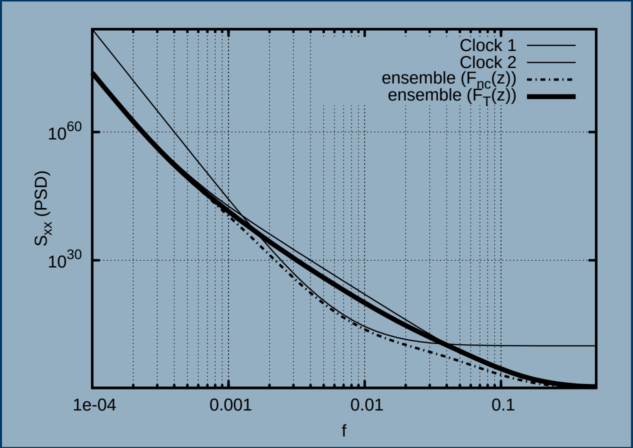
Finite-lag WF

Example #2 – finite-lag WF, $T = 1\,T_s$



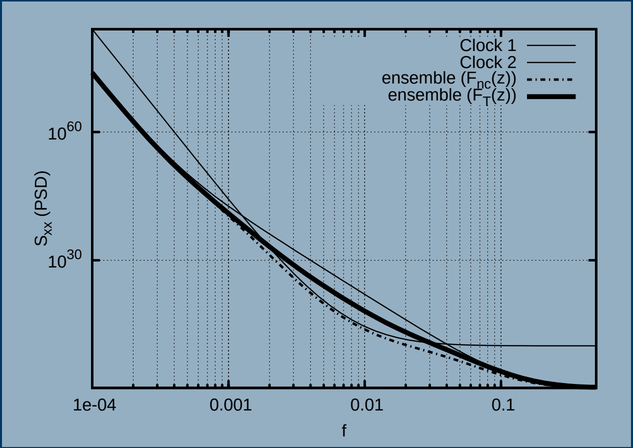
Finite-lag WF

Example #2 – finite-lag WF, $T = 30\,T_s$



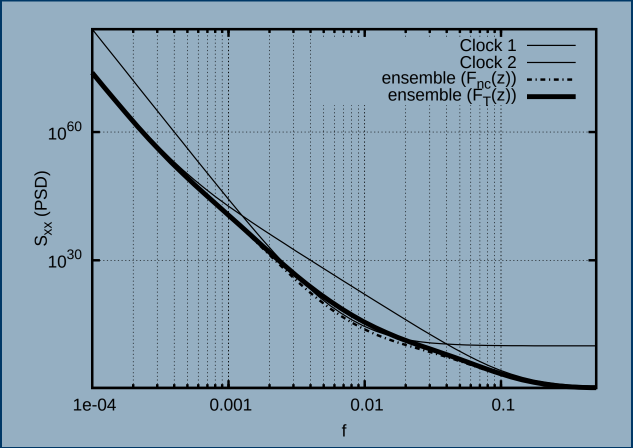
Finite-lag WF

Example #2 – finite-lag WF, $T = 70\,T_s$



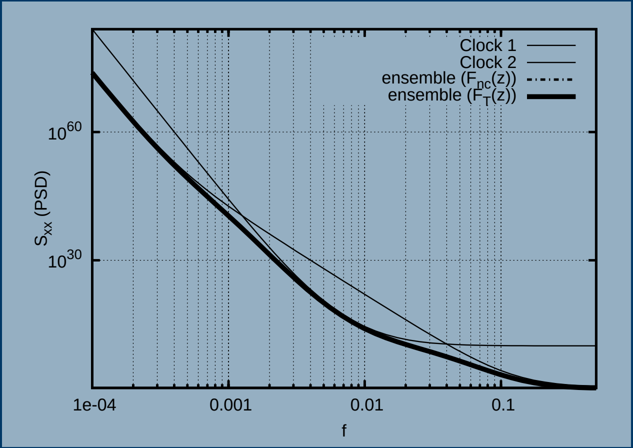
Finite-lag WF

Example #2 – finite-lag WF, $T = 150\,T_s$



Finite-lag WF

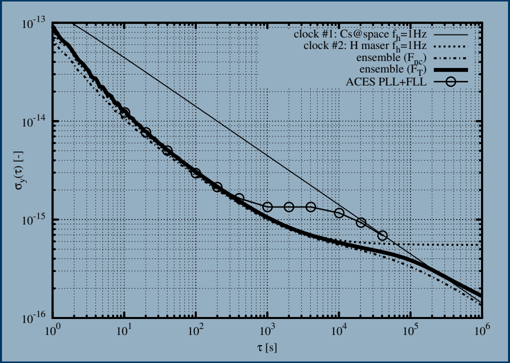
Example #2 – finite-lag WF, $T = 300\, T_s$



WF performance on real example

Atomic Clock Ensemble in Space

Example #3 – Atomic Clock Ensemble in Space (ACES) model



- current solution based on PLL & FLL
- finite-lag WF is better & substantially simpler

Conclusion

- given optimal linear estimator for stationary ensemble
- $F_T(z)$ may be significant improvement over $F(z)$
- do not feedback, estimate wherever possible
- save raw data – allow multiple different $F_T(z)$
- outlook
  - unify with KF approach, generalize for $N > 2$