Scheduling

- set **T** of ***n*** **tasks** **T** $=\{T_1, T_2, \ldots, T_n\}$
- set **P** of ***m*** **processors** **P** $=\{P_1, P_2, \ldots, P_m\}$
- set of additional resources

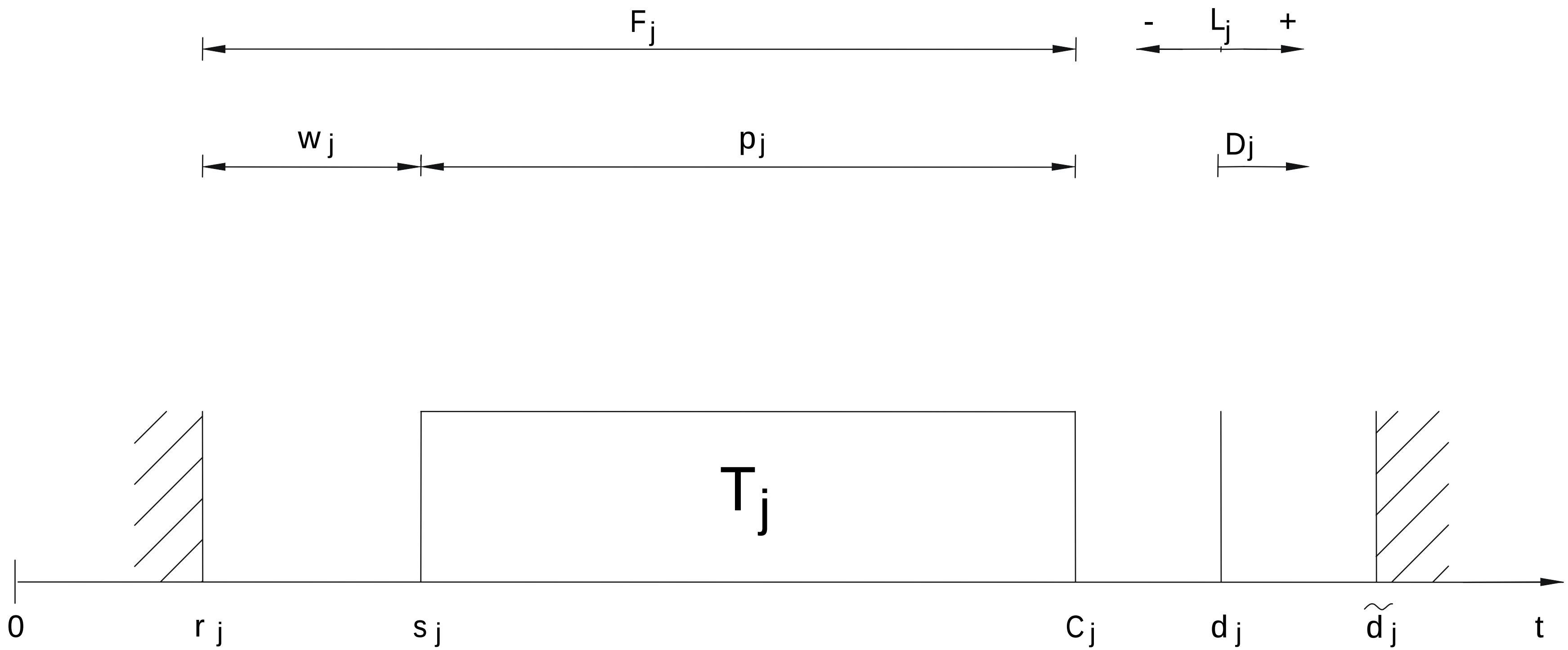
Scheduling = **assignment of task to processors** in order to complete the tasks under imposed constraints

- on line / off line
- deterministic parameters – combinatorial **optimization** algorithms
- schedules are represented by **Gantt charts**

Basic constraints

- each task is to be processed by at most **one processor** at a time
- each processor is capable of processing at most **one task** at a time
- task T_j is processed in time interval $[r_j,\infty)$
- all tasks are **completed**
- if tasks T_i , T_j are in the relation $\boldsymbol{T_i < T_j}$, the processing of T_j is not started before completion of T_i
- in the case of **non preemptive** scheduling no task is preempted, otherwise the number of preemptions if **finite**
- additional resource constraints, if any, are satisfied

Task parameters



Task parameters

- release (arrival, disponability) time $\boldsymbol{r_j}$
- start time $\boldsymbol{s_j}$, completion time $\boldsymbol{C_j}$
- due date $\boldsymbol{d_j}$
- deadline $\boldsymbol{\tilde{d}_j}$ is hard real time limit by which T_j must be completed
- waiting time $\boldsymbol{w_j}$
- processing time $\boldsymbol{p_j}$
- flow time $\boldsymbol{F_j = C_j - r_j}$
- lateness $\boldsymbol{L_j = C_j - d_j}$
- tardiness $\boldsymbol{D_j = max\{C_j - d_j, 0\}}$

Standard notation $\alpha/\beta/\gamma$ by Graham

$\alpha_1 =$	…	1 processor	
	P	parallel identical processors	
	Q	parallel uniform proc.	$p_{ij} = p_j/b_i$ (b_i is proc. speed)
	R	parallel unrelated proc.	p_{ij} is arbitrary
	O	dedicated "open-shop"	
	F	dedicated "flow-shop"	
	J	dedicated "job-shop"	
$\alpha_2 =$	…	variable number of processors	
	k	given number of processors	

Tasks: $\beta = \beta_1 \beta_2 \beta_3 \beta_4 \beta_5 \beta_6 \beta_7 \beta_8$

$\beta_1 \in \{\ldots, \text{pmtn}\}$	preemption
$\beta_2 \in \{\ldots, \text{res}\}$	additional resources
$\beta_3 \in \{\ldots, \text{prec,tree,chain}\}$	precedence constraints
$\beta_4 \in \{\ldots, r_j\}$	release time
$\beta_5 \in \{\ldots, p_j=k, p_L \leq p_j \leq p_U\}$	var/const/limited proc.time
$\beta_6 \in \{\ldots, \tilde{d}\}$	deadline
$\beta_7 \in \{\ldots, n_j \leq k\}$	limited number of jobs in Job-shop
$\beta_8 \in \{\ldots, \text{no-wait}\}$	buffers with infinite/zero capacity

Optimality criterion γ

$\gamma \in \{C_{max}, \Sigma C_j, \Sigma w_j C_j, L_{max}, \ldots\}$

$C_{max} = \max_{\forall j} \{C_j\}$

$L_{max} = \max_{\forall j} \{C_j - d_j\}$

For example standard notation $P||C_{max}$ means:

variable number of identical parallel processors, non-preemptive, no precedence constraints, all tasks starting at time 0, variable processing time, make spawn optimization