Continuous Petri Nets and Polytopes

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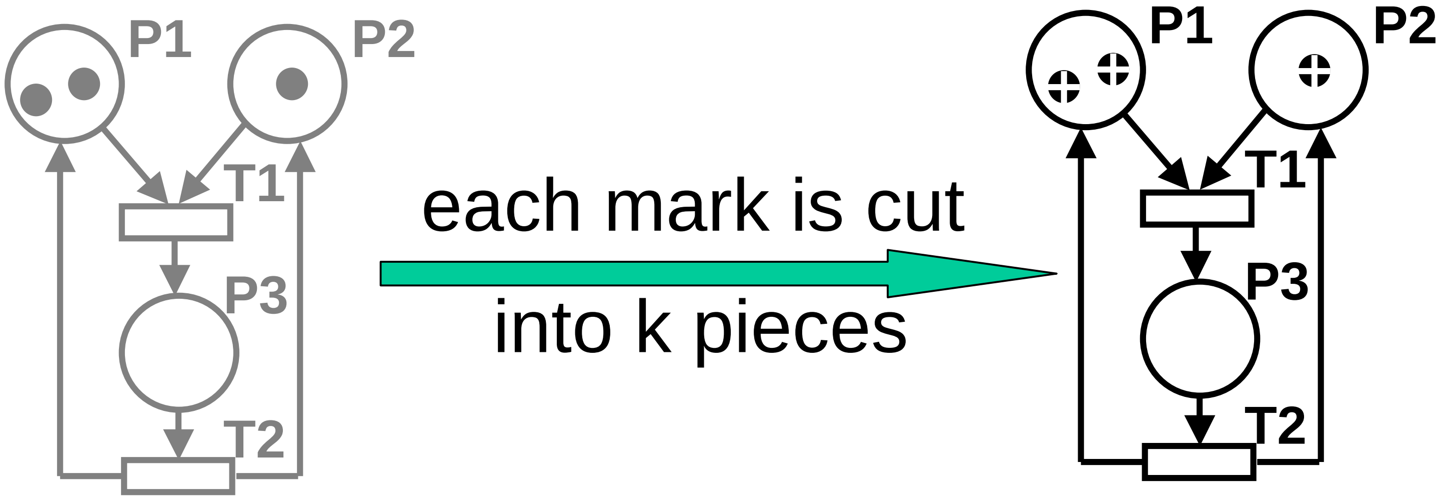
From autonomous PN …….. to autonomous CPN



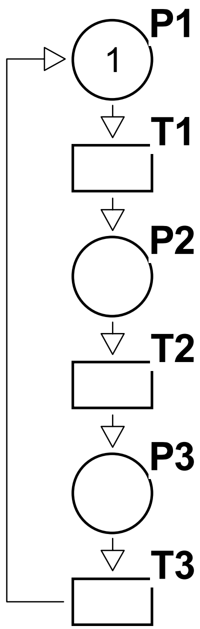
$$\begin{bmatrix}2\\1\\0\end{bmatrix} \underset{\overset{T2}{\longleftarrow}}{\overset{T1}{\longrightarrow}} \begin{bmatrix}1\\0\\1\end{bmatrix}$$

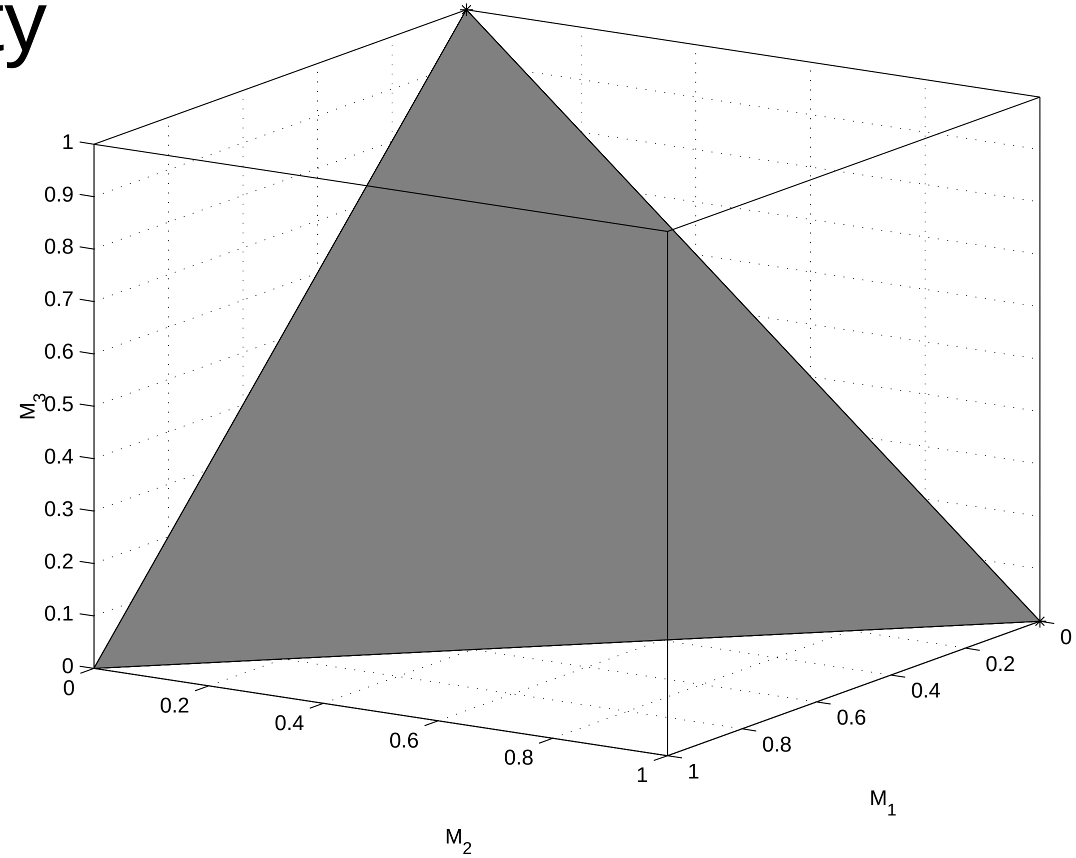
$$\begin{bmatrix}2\\1\\0\end{bmatrix} \underset{\overset{T2}{\longleftarrow}}{\overset{T1}{\longrightarrow}} \begin{bmatrix}2-1/k\\1-1/k\\1/k\end{bmatrix} \underset{\longleftarrow\cdots}{\cdots\longrightarrow} \begin{bmatrix}1+1/k\\1/k\\1-1/k\end{bmatrix} \underset{\overset{T2}{\longleftarrow}}{\overset{T1}{\longrightarrow}} \begin{bmatrix}1\\0\\1\end{bmatrix}$$

- no threshold
- modeling of continuous systems / approximation of DES

Continuous state space

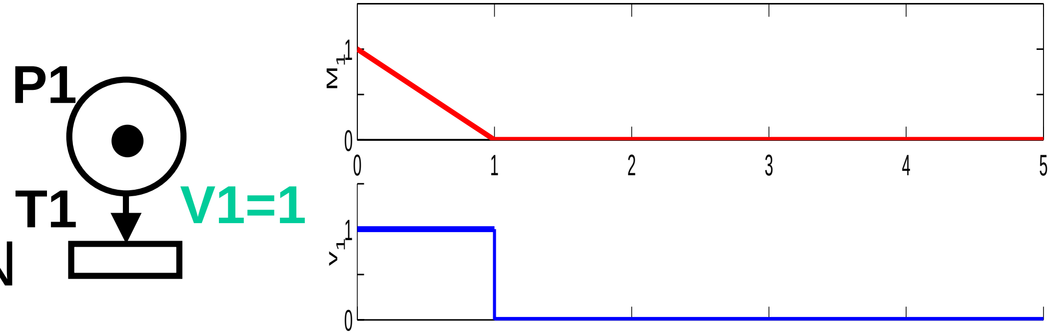
- the state space is continuous when k tends to infinity



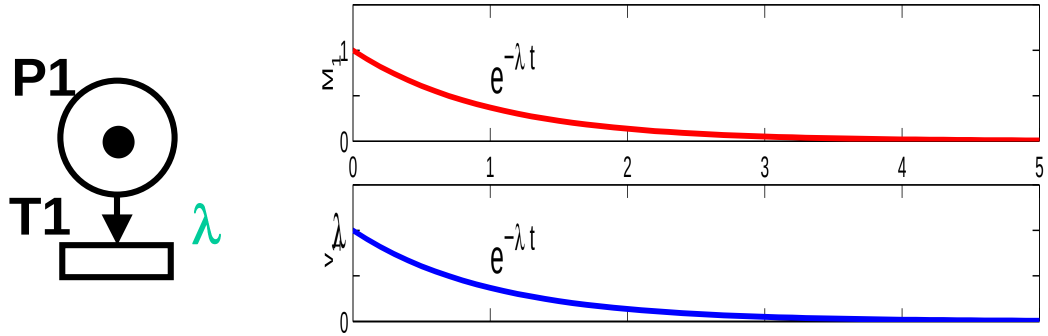


3 Timed Continuous Petri Nets models

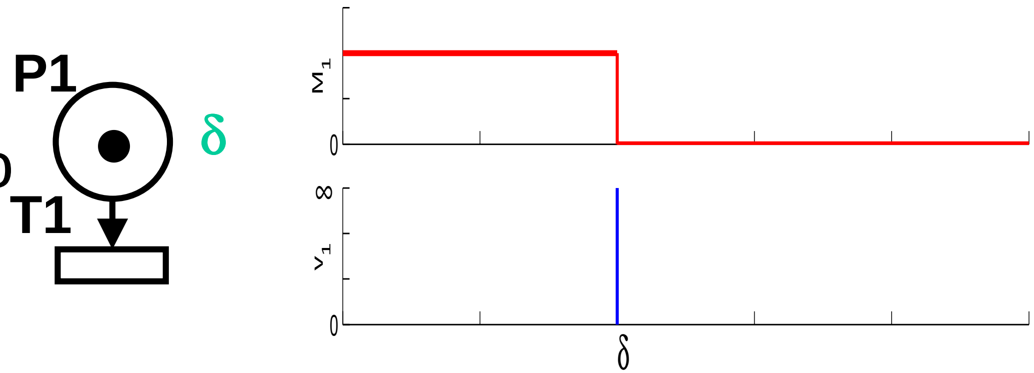
1. David&Alla
   - speed restriction
   - further called CCPN



2. Recalde&Silva
   - differential equations



3. Cohen et al.
   - delays associated to places



CCPN - Constant speed continuous PN

$R = [P, T, V, Pre, Post, M(0)]$

- P, T as in usual PN
- Pre, Post: $P \times T \rightarrow R_0^+$
- M(0): $P \rightarrow R_0^+$ M(t) is marking at time t
- V: $T \rightarrow R_0^+$ $v_j(t) \in \langle 0, V_j \rangle$ is instantaneous firing **speed**

T_j is **strongly enabled** if $\forall P_i \in {}^\circ T_j$; $M_i > 0$

P_i is **supplied** $\exists T_j \in {}^\circ P_i$; T_j is strongly or weakly enabled

T_j is **weakly enab.** if ($\exists P_i \in {}^\circ T_j$; $M_i = 0$ and P_i is supplied) and ($\forall P_k \in {}^\circ T_j$; $P_k \neq P_i$; $M_i > 0$ or P_k is supplied)

CCPN - Constant speed continuous PN (cont.)

Recursively defined supplied places and weakly enabled transitions ere determined by **iterative algorithm** with proven convergence.

Balance of the place P_i is:

$$B_i(t) = \sum_{T_j \in {}^\circ P_i} Post(P_i, T_j) \cdot v_j(t) - \sum_{T_j \in P_i^\circ} \Pr e(P_i, T_j) \cdot v_j(t)$$

B_i corresponds to the **derivative** of P_i marking:

$$m_i(t + dt) = m_i(t) + B_i(t) \cdot dt$$

Algorithms calculating instantaneous firing speed:

[DA98] … based on **iterative** approach

this article …**analytical** determination of subspace of possible instantaneous firing speeds.

Free speed model

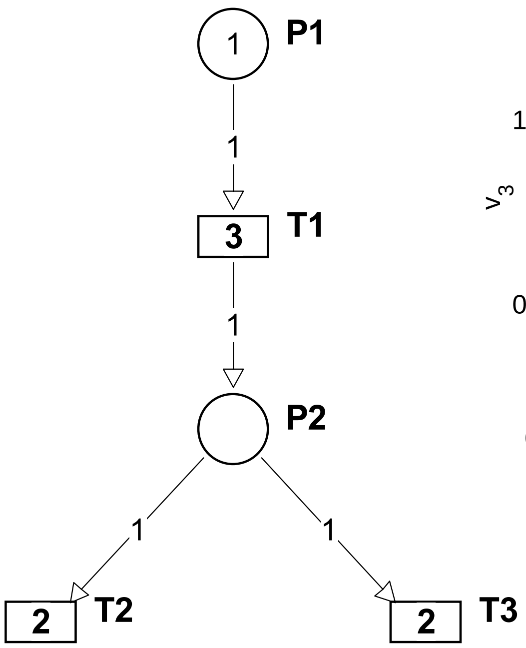
- CCPN **without speed maximization** (this model does not work in earliest firing mode)
- in the limit case ($\forall T_j$; $V_j=\infty$) this model is identical to **autonomous** CPN
- system of inequalities (no difference strongly/weakly en.) defines **polytop** Π

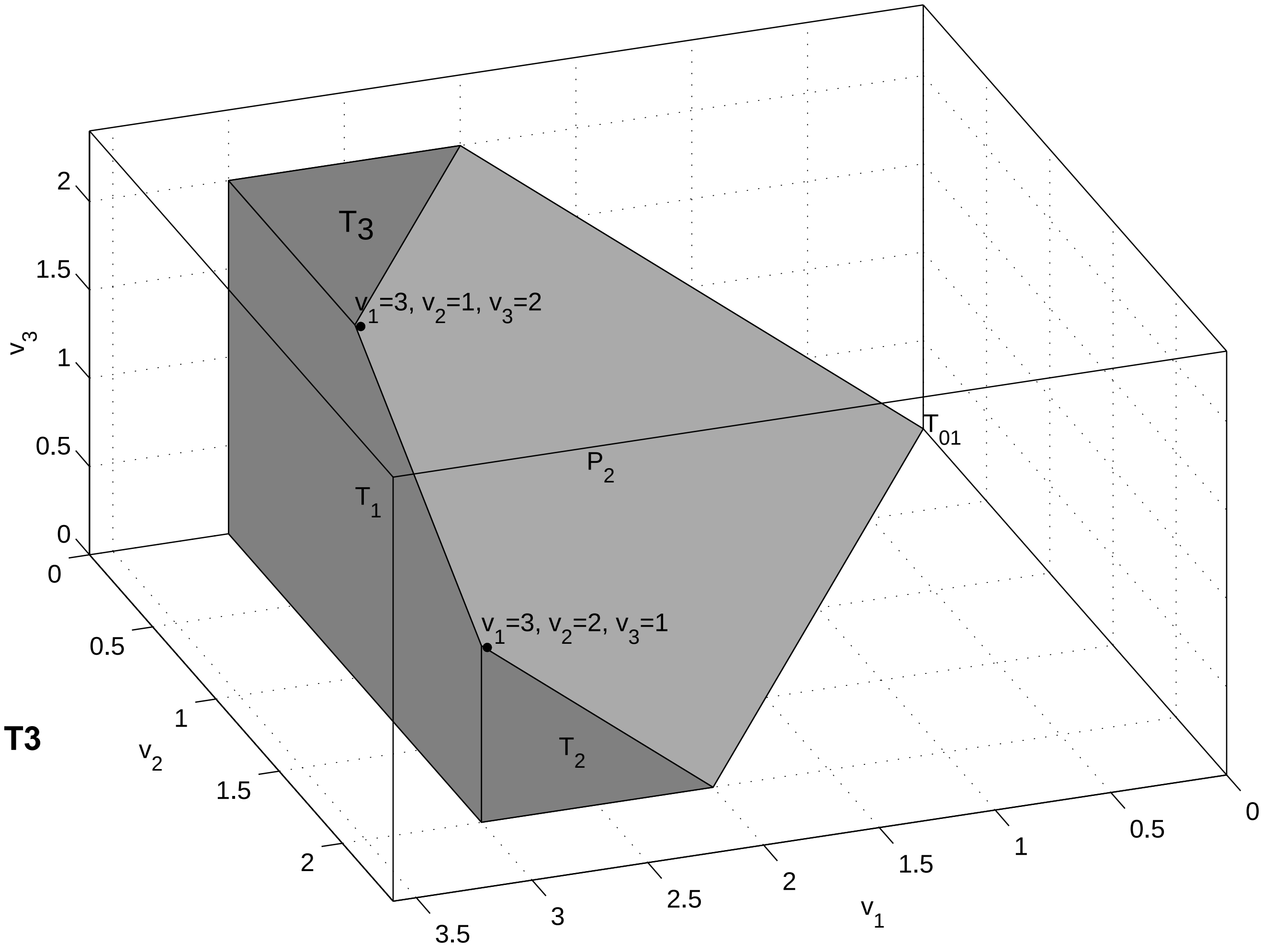
$v_j(t) \le V_j \;\; \forall$ enabled T_j

$v_j(t) \ge 0 \;\; \forall$ enabled T_j

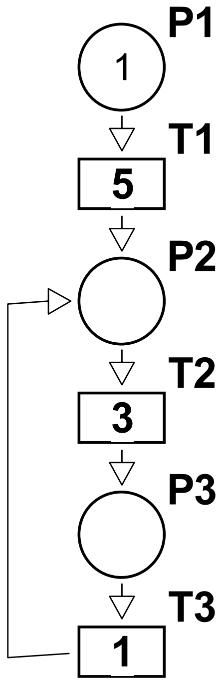
$B_i(t) \ge 0 \; \forall \; P_i$; $\exists$ enabled $T_j \in P_i^\circ$ **and** $M_i=0$

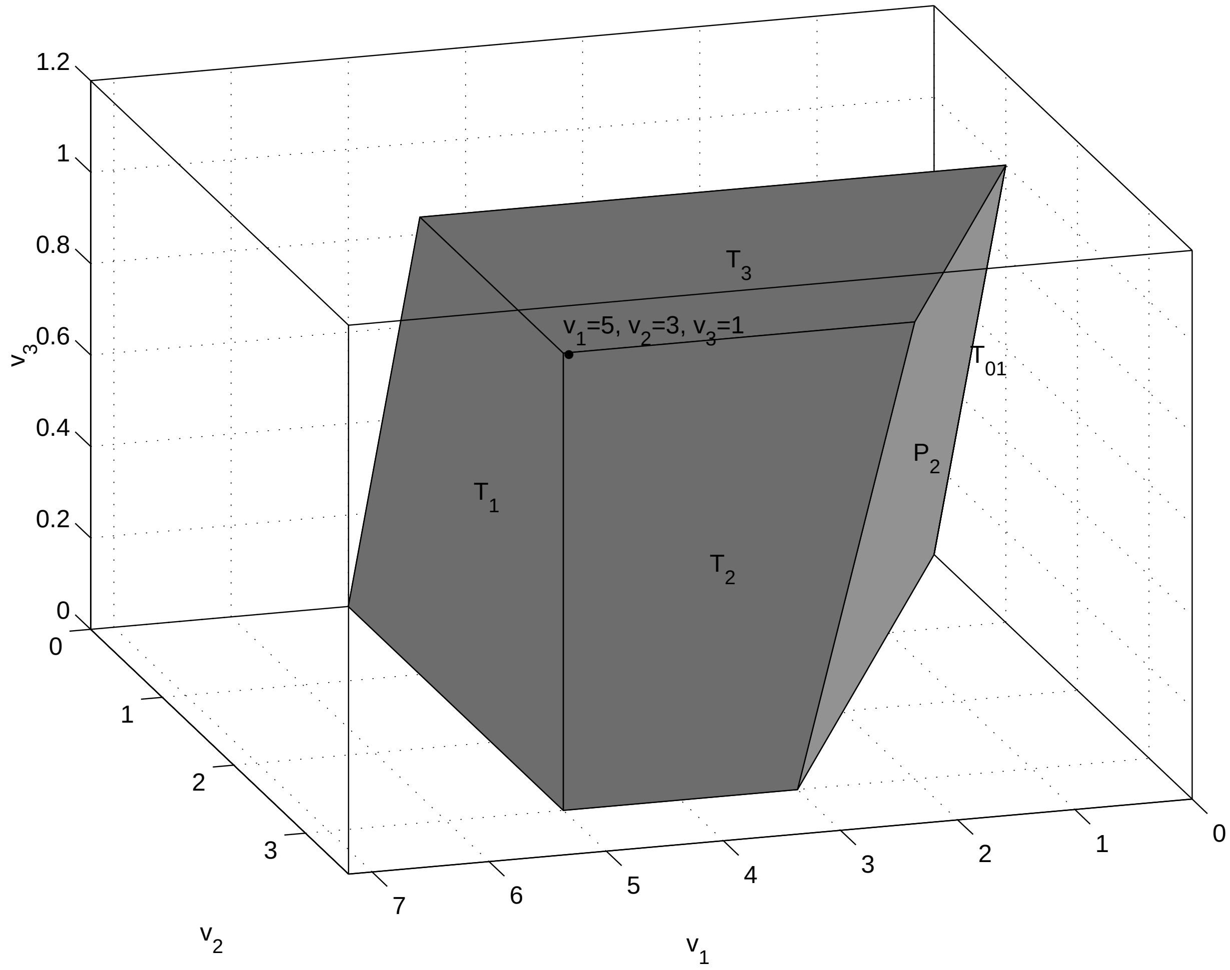
Free speed model – example of conflict





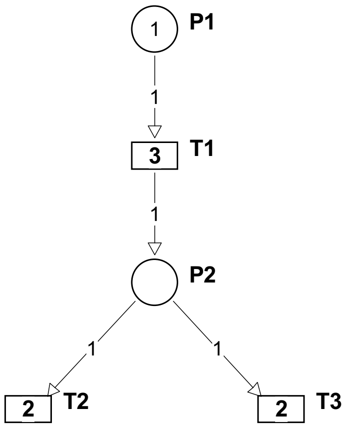
Free speed model – empty loop paradox

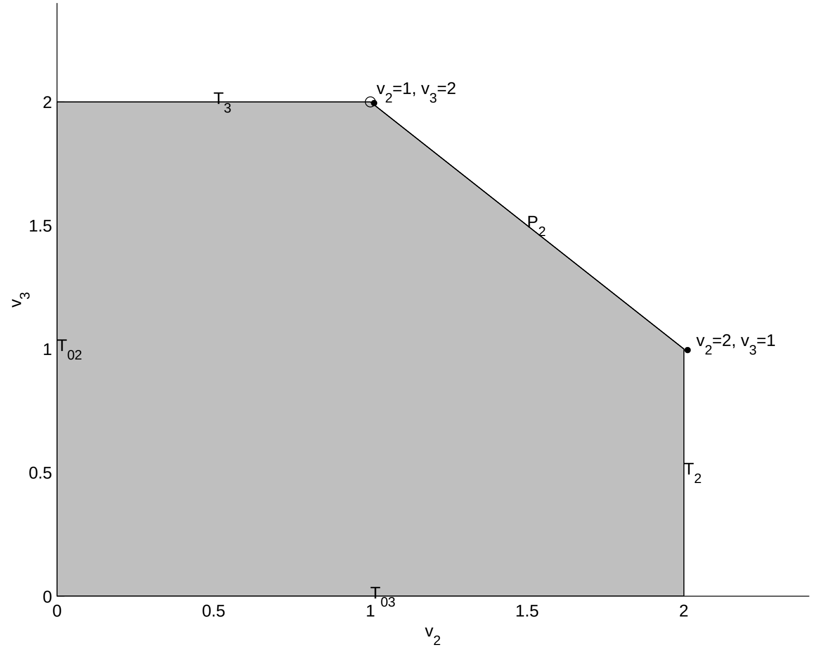




Maximum speed model ~ CCPN

reduction of polytop Π (polytop dimension ~ number of weakly enabled transitions)





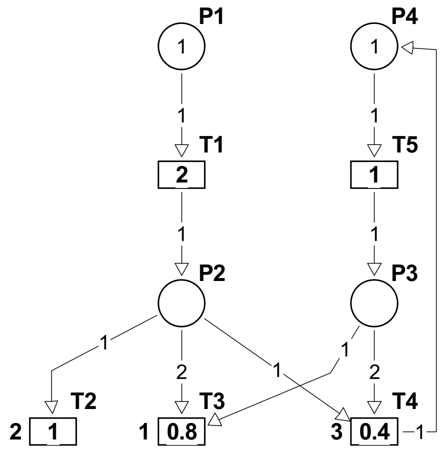
Definition: G, the maximum speed area, is the subset of polytop Π such that for each speed $v \in G$ there does not exist any other speed $u \in \Pi$ such that $u_k \geq v_k \ \forall \ k=1\ldots d$

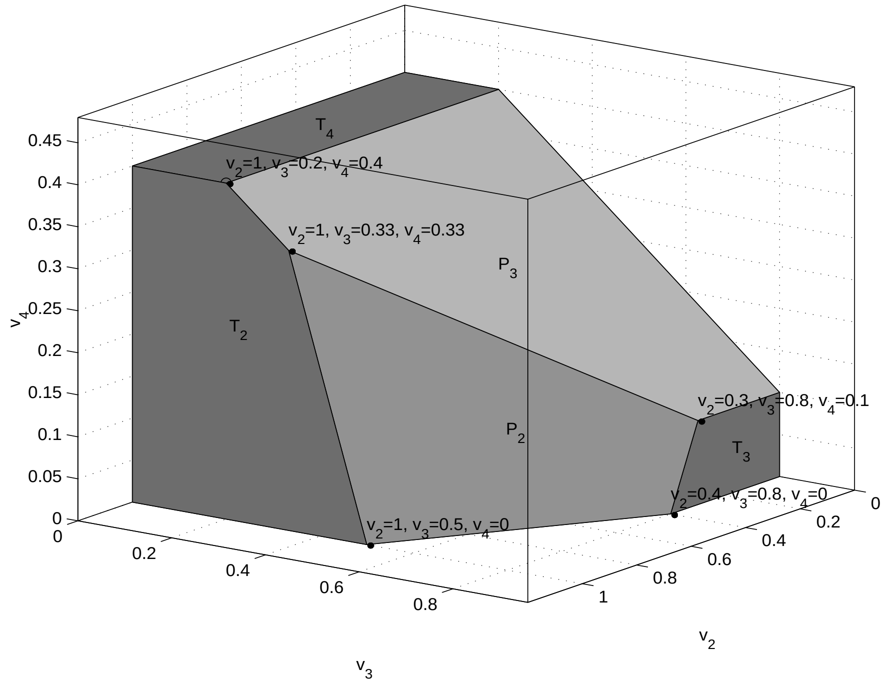
- **face belongs to G** $\Leftrightarrow$ all its vertices belong to G i.e. $F_k \in G \Leftrightarrow \forall \ F_0 \in F_k ; F_0 \in G$

Actual conflict

Definition: actual conflict between T_j and T_k exists if $\exists$ structural conflict and $\exists$ $v, v' \in G$; such that $v_j < v'_j$ and $v_k > v'_k$

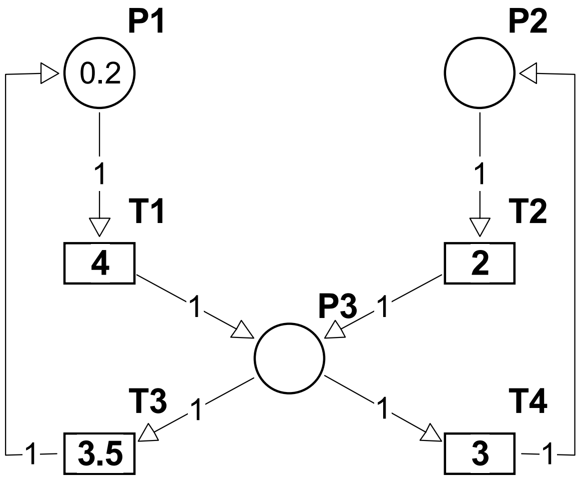
- actual conflict exists $\Leftrightarrow$ there exists face $F_k \in G$; $k \geq 1$
- actual conflict does not exist $\Leftrightarrow$ G is exactly one vertex $\Rightarrow$ one call of **LP** determines G in polynomial time
- **G is not convex:**

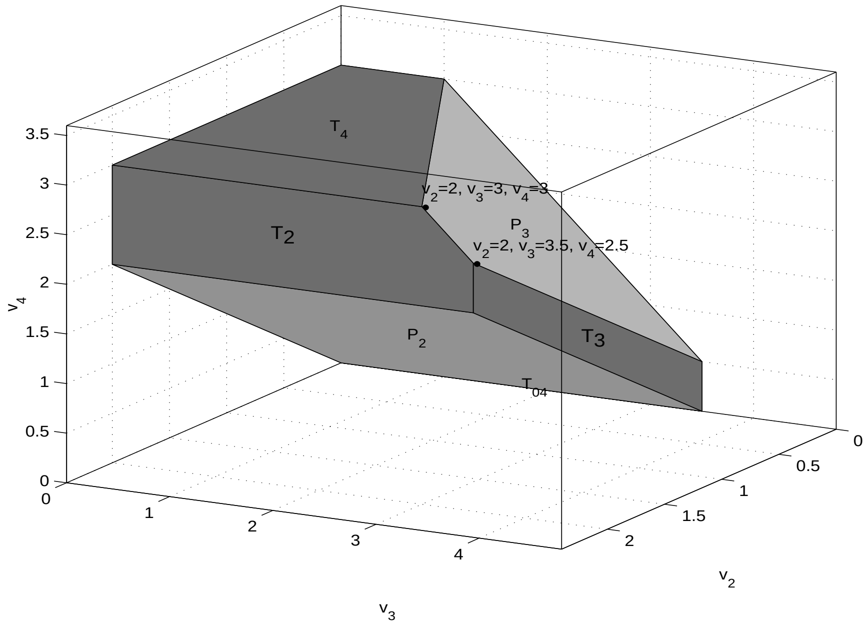




Resolution of actual conflicts by priorities

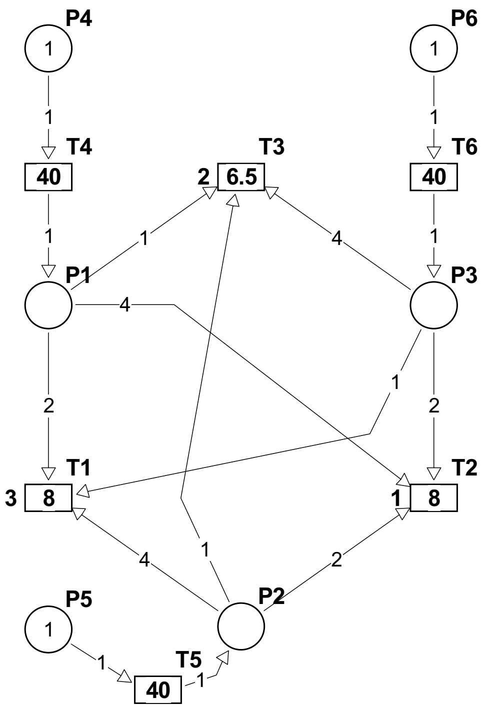
Definition: H, the priority determined area, is the subset of G such that for each speed $v \in H$ and for any other speed $u \in G$ and for any T_j such that $v_j < u_j$ there exists some T_k such that priority(T_k) $\geq$ priority(T_j) and $v_k > u_k$ (If exists coordinate $v_j < u_j$ then we find always another coordinate $v_k > u_k$ with higher or equal priority)

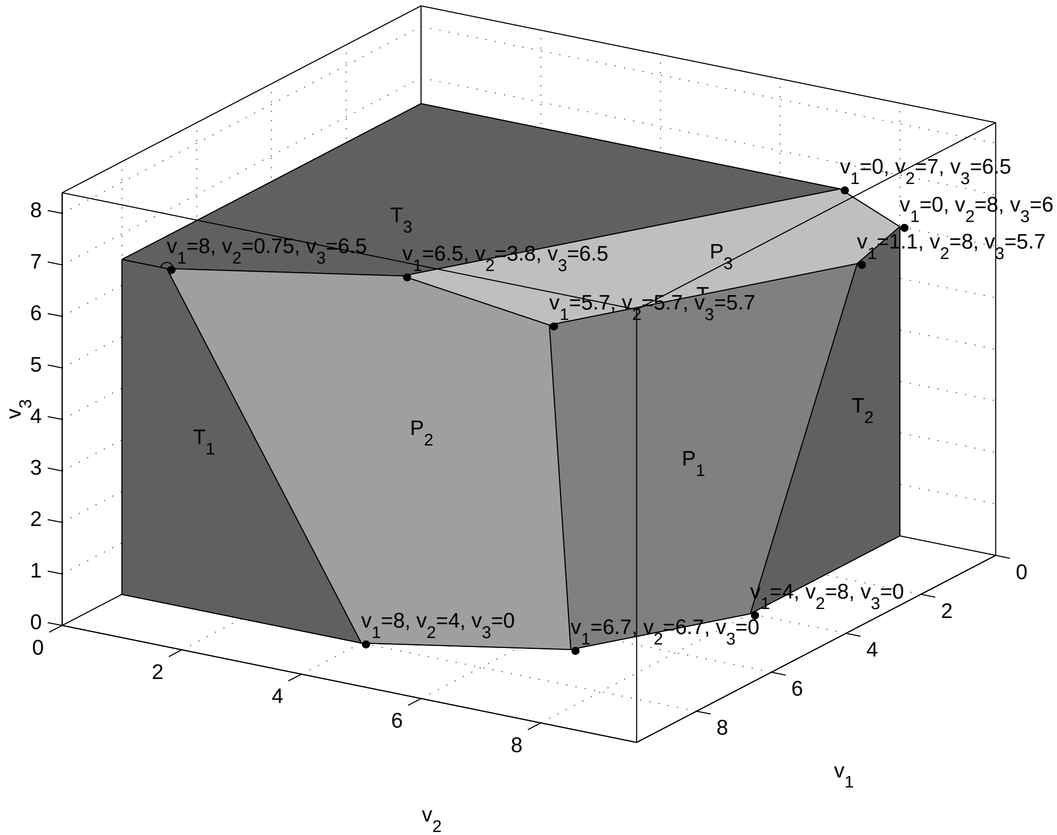




Example of area G and area H

- 9 vertices (2^3 priority combinations +1)
- G is not convex





Algorithms for instantaneous firing speed

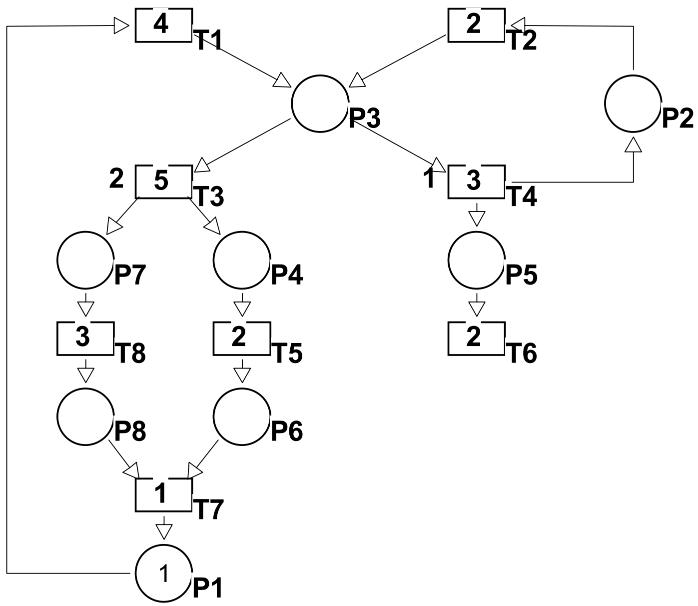
- vertex enumeration (actual conflict resolution not needed)
    1. vertex representation of polytop
    2. selection of vertices belonging to G
    3. selection of vertices belonging to H
- Linear Programming (polynomial time complexity)

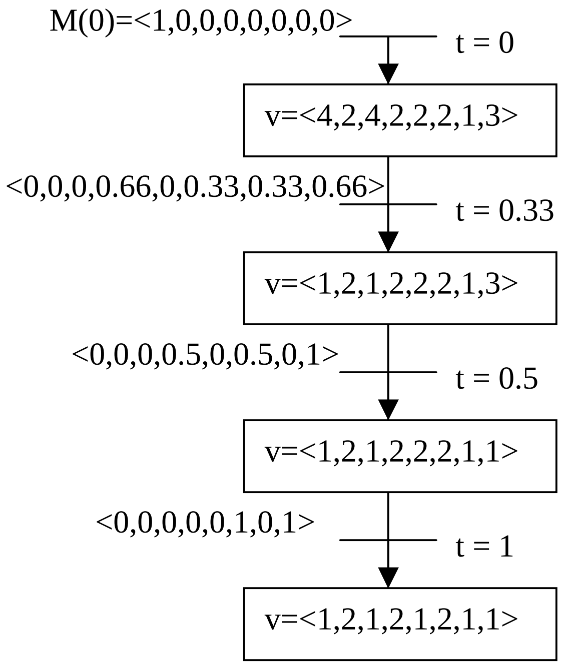
    for each priority level k

    1. $J_j = 1$ for each j on this priority level otherwise $J_j = 0$
    2. find solution by Linear Programming
    3. add one equation per each j on this priority level

Evolution graph

- one polytop with determined speed per each IB state





Conclusions

- CCPN speed determination when actual conflicts are present
- vertex enumeration (polytop, G, H) and LP solution (polynomial)
- occurrence of 0^+ marking in initial IB state – free speed model of empty circuit
- $H \subset G$, i.e. speed maximization is prior to priority resolution (weak priority)