Complexity of Consistency and Complete State Coding for Signal Transition Graphs

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Overview

- A brief recalling of the report from ES-Colloquium 10/2005 on (the following work with colleagues from Univ. Augsburg)
  - Schäfer M., Vogler W., Jančar P.: Determinate STG Decomposition of Marked Graphs; in Proceedings 26th Int. Conf. on Application and Theory of Petri Nets and Other Models of Concurrency (ICATPN 2005), Miami, FL, June 20-25, 2005, Lecture Notes in Computer Science, Vol. 3536, Springer Verlag 2005, p. 365 - 384
- A more detailed report on (the following work with colleagues from Univ. Stuttgart)
  - Esparza J., Jančar P., Miller A.: On the Complexity of Consistency and Complete State Coding for Signal Transition Graphs; in Proceedings 6th International Conference on Application of Concurrency to System Design (ACSD 2006), Turku, Finland, June 2006, IEEE Computer Society 2006, pp. 47–56

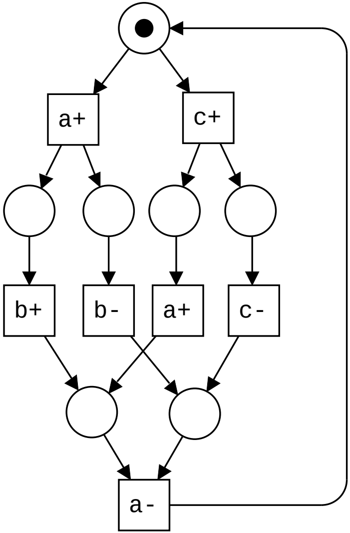
Outline

- Asynchronous circuits.
- Signal transition graphs.
- Consistency problem.
- Complete State Coding (CSC) problem,
  Unique State Coding (USC) problem.
- Polynomiality of consistency for Marked Graph STGs (MG-STGs).
- Co-NP completeness of CSC and USC problems (for 1-bounded acyclic MG STGs and) for 1-bounded live MG STGs.
- Some additional results.

Asynchronous circuits

- For implementation of state dependent circuits
- No clock signal
- Communication with **signal edges** (a^+ raising, a^- falling)
- Distinction between
  - input signals (controlled by the environment)
  - output signals (controlled by the circuit)
- Advantages
  - Average case efficiency instead of worst case efficiency
  - Reduced power consumption
  - Very low electromagnetic emission
- Disadvantage
  - Complex synthesis

A (free-choice) STG



Signal transition graphs - definition

STG: a Petri net based specification of the behaviour of an asynchronous circuit (under some assumptions on the environment).

$A = \{a_1, a_2, \ldots, a_n\}$ ... a set of signals
$\mathcal{L} = \{a_1^+, a_1^-, a_2^+, a_2^-, \ldots, a_n^+, a_n^-\}$ ... the set of (transition) labels

STG: $S = (N, M_0, \ell)$, where

- (N, M_0) is a Petri net, $N = (P, T, F)$, and
- $\ell : T \longrightarrow \mathcal{L}$

An STG S is implementable if there exists a state coding mapping

$$\lambda : Reach(M_0) \longrightarrow \{0, 1\}^n$$

($\lambda(M)$ gives the signal values of the corresponding circuit state) which is consistent and has the CSC (or stronger USC) property.

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Consistency (a characterization)

An STG $S = (N, M_0, \ell)$ is inconsistent
(i.e., it admits no consistent state coding mapping)
iff there is

a pair (M, a)
where $M_0 \longrightarrow^* M$ and a is a signal

such that one of the following conditions holds:

(1) M enables ua^+ and va^-
for some a-free sequences u, v,

(2) M enables a^+ua^+ or a^-ua^-
for some a-free sequence u,

(3) M is reachable by w_1a^+u and by w_2a^-v
for some a-free sequences u, v (and some w_1, w_2).

Complete State Coding (CSC) and Unique SC (USC)

The signal set $A = \{a_1, a_2, \ldots, a_n\}$ is partitioned into input signals and output signals.

A state coding mapping $\lambda : Reach(M_0) \longrightarrow \{0,1\}^n$ has the CSC property if (it is consistent and) it satisfies:

when $\lambda(M) = \lambda(L)$ for two reachable markings $M \neq L$ then M and L enable exactly the same output signals.

λ has the USC property if it is injective ($\lambda(M) \neq \lambda(L)$ for $M \neq L$).

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Consistency for Marked Graph STGs (MG STGs)

$S = (N, M_0, \ell)$ is an MG STG if
N is a marked graph (net), i.e.,
each place has at most one input and at most one output transition.

We note: (the set of places of) any cycle in an MG is a trap (if marked, it cannot get unmarked) and a siphon (if unmarked, it cannot get marked)

Consistency problem for MG STGs:
Instance: an MG STG $S = (N, M_0, \ell)$
Question: does S admit a consistent state coding mapping ?

We recall:
boundedness, liveness
(any live and bounded MG is strongly connected)

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Polynomiality of consistency for MG STGs

J. Esparza at ACSD'03:
Consistency is polynomial for live, bounded, and cyclic free-choice STGs

(here 'cyclic' means: the initial marking can always be (re)reached)
(recall that 'free-choice' is the net property guaranteeing that if some output transition of a place is enabled then all its output transitions are enabled)

Remark:

- it is still open if polynomial without cyclicity
- liveness is important: consistency is PSPACE-complete for 1-bounded free choice STGs (as we show here)

Theorem: The consistency is polynomial for (all) MG STGs.

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Ideas for polynomiality of consistency for MG STGs

We call a (N, M_0) normalized if every cycle of N is marked at M_0.

Claim. Let (N, M_0) be a normalized marked graph, where C_N is the incidence matrix of N.
An integer vector $X_0 \geq 0$ is a solution of the inequation $M_0 + C_N \cdot X \geq 0$ if and only if it is the Parikh vector of a transition sequence σ such that $M_0 \xrightarrow{\sigma}$. (If $M_0 \xrightarrow{\sigma} M$ then $M_0 + C_N \cdot X_0 = M$.)

Claim (a fact from linear programming).
Given C_N where every row contains at most one $+1$ and at most one -1 (which is the case for MG),
for any linear objective function $f(X)$ the optimal solution X_0 of the inequations $X \geq 0$, $M_0 + C_N \cdot X \geq 0$ (if it exists) is integer,
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Ideas for polynomiality of consistency for MG STGs - ctd.

Proposition. A marked graph STG $S = (N, M_0, \ell)$ is inconsistent iff one of the following conditions holds:

(1') there is a reachable M ($M_0 \rightarrow^* M$) such that
$M \xrightarrow{a^+}$ and $M \xrightarrow{a^-}$ for some signal a,

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The previous claims enable deciding these conditions by using (usual) linear programming.

E.g.: maximize $f(X)$ subject to $X \geq 0$, $M_0 + C_N \cdot X \geq 0$
where

$$f(X) = \sum_{t \in \ell^{-1}(a^+)} X(t) - \sum_{t \in \ell^{-1}(a^-)} X(t)$$

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Co-NP-completeness of USC and CSC for MG STGs

Theorem.

- The CSC problem and the USC problem are co-NP-hard for (1-bounded acyclic marked graph STGs and for) 1-bounded live marked graph STGs.
- The CSC problem and the USC problem for live and bounded free-choice STGs are in co-NP.

We show the main technical lemma (of the NP-hardness part); a transition sequence is balanced if the numbers of occurrences of labels a^+ and a^- are the same for each signal a:

Lemma. The following problem is NP-complete:
Instance: a (consistent) STG $S = (N, M_0, \ell)$ such that (N, M_0) is a 1-bounded, acyclic marked graph.
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The crucial point: NP-hardness

Given a boolean formula φ in CNF, we construct a 1-bounded acyclic MG STG S_φ so that

$$\varphi \text{ is satisfiable} \iff S_\varphi \text{ admits } M_0 \xrightarrow{\sigma} M_1 \xrightarrow{\tau} M_2$$
$$\text{for some nonempty balanced } \tau.$$

Example. $\varphi \equiv (x_1 \vee \overline{x}_2 \vee x_3) \wedge (x_1 \vee x_2 \vee \overline{x}_3)$

Signals of S_φ :

- x_1, x_2, x_3 (variables)
- c_1, c_2 (clauses)
- $p_{1,1}$, $p_{1,2}$, $p_{2,2}$, $p_{3,1}$ (x_i positively in c_j)
- $n_{2,1}$, $n_{3,2}$ (x_i negatively in c_j)
- special (auxiliary) signals $S_0, S_1, S_2, \ldots$ ('brackets')

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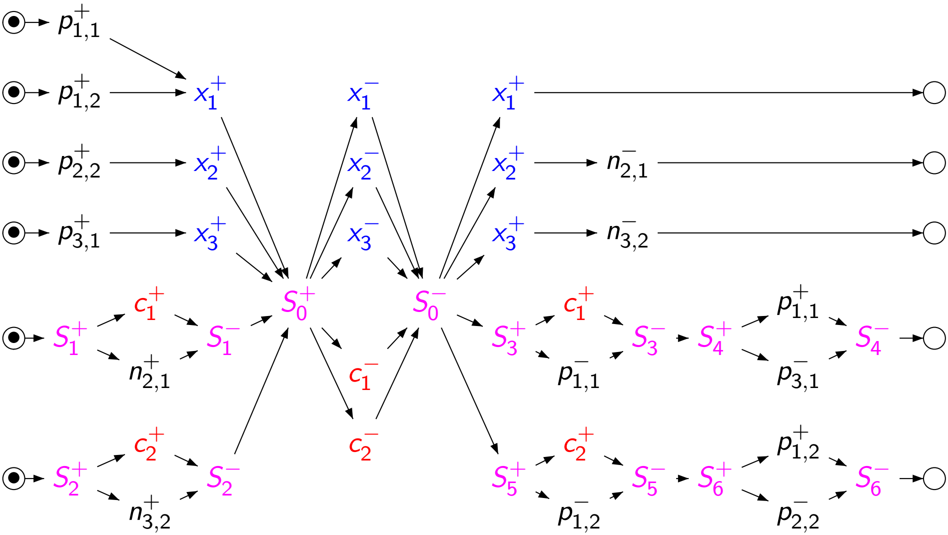

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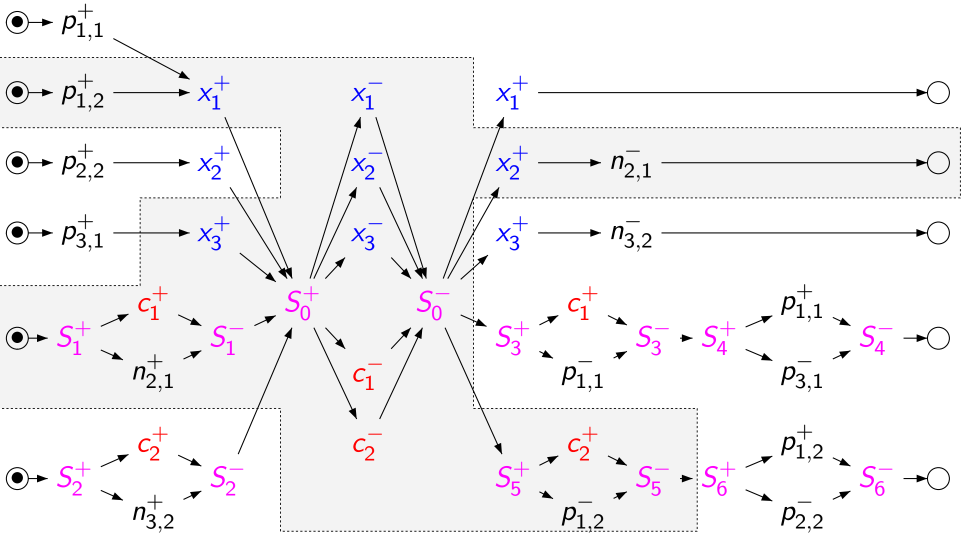

From 1-bounded acyclic MG to 1-bounded live MG

We define a new STG S'_φ by adding a 'final segment' to S_φ: we add a fresh signal f and construct a 'linear' net N_f with the behaviour

$$f^+ \; \ell_1 \; \ell_2 \; \cdots \; \ell_k \; f^-$$

so that S'_φ is a live 1-bounded MG STG and the set of all its transitions is balanced ...

CSC and USC in co-NP for live and bounded FC STGs

Using of a result by Yamasaki, Huan, Murata (2001):

Theorem.
Let (N, M_0) be a live and bounded free-choice Petri net, and let C_N be its incidence matrix.
An integer vector $X_0 \geq 0$ is the Parikh vector of a transition sequence enabled at M_0 if and only if

1. $M_0 + C_N \cdot X_0 \geq 0$, and
2. $M = M_0 + C_N \cdot X_0$ marks all (nonempty) traps of N_{X_0}.

For a net $N = (P, T, F)$ and $X : T \rightarrow \mathbb{N}$, we denote by $N_X = (P_X, T_X, F_X)$ the subnet of N defined as follows: T_X is the set of transitions of T for which $X(t) \geq 1$, $P_X = {}^\bullet T_X \cup T_X^\bullet$, and F_X is the projection of F on $(P_X \times T_X) \cup (T_X \times P_X)$.

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CSC and USC in co-NP for LB FC STGs - ctd.

S does not have the CSC property if and only if

there are sequences u_1, u_2 such that

- $M_0 \xrightarrow{u_1} M_1$, $M_0 \xrightarrow{u_2} M_2$,
- $M_1 \neq M_2$,
- for each signal a:

$$P(u_1)(a^+) - P(u_1)(a^-) = P(u_2)(a^+) - P(u_2)(a^-)$$

- M_1, M_2 enable different output signals

A nondeterministic polynomial algorithm (to check the above condition)

1. make several (straightforward) guesses
2. create a corresponding system of linear inequalities,
3. guess an integer (candidate for) solution of polynomial size,
4. check that it is indeed a solution.

(Variables for transition sequences are replaced by variables for their Parikh vectors. The 'trap problem' can be handled by guessing N_X and a subset of its places not containing a trap ...)

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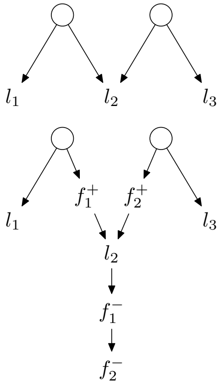
General bounded STGs

Proposition. The consistency problem, the CSC problem and the USC problem are PSPACE-complete for k-bounded STGs (for any fixed k).

Free choice without liveness does not help:

Transforming a 1-bounded STG into a 1-bounded free-choice STG while keeping (in)consistency



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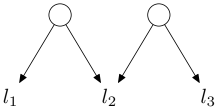

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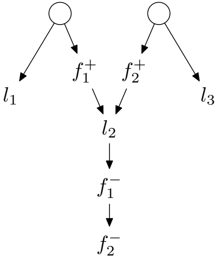

General STGs

Proposition. The consistency problem and the CSC problem for general STGs are decidable but EXPSPACE-hard.

(the reachability problem for Petri nets ...)

Some open problems

- Is the CSC (USC) problem polynomial on MG STGs with injective labelling of transitions ?
- Is consistency polynomial for live and bounded free-choice STGs ?