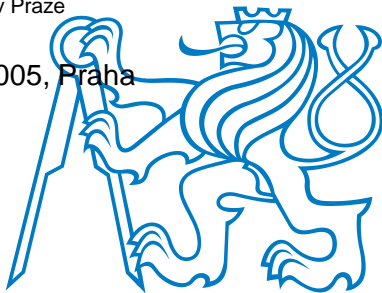


# Řízení provozu na světelné křižovatce

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# Outline

## Intersection Model

- Queue Modeling

- Intersection Model

- State Space and Linearization

## Intersection Control

- LQ Controller

- MPC Controller

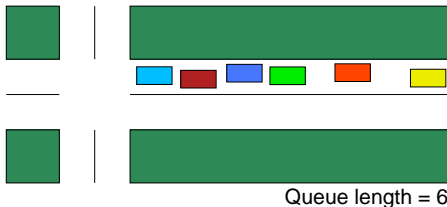
- Comparing the Results

## Simulation Model

## Summary

# Queue Modeling

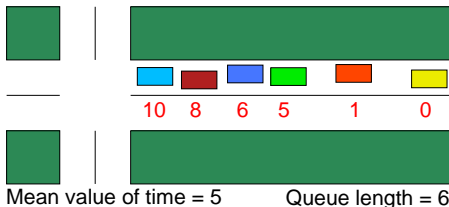
- ▶ Commonly used queue model:
  - ▶ Only the queue length is used



- ▶ Model does not include information about the queue dynamics

# Queue Modeling

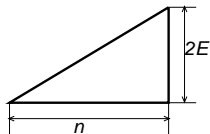
- ▶ Commonly used queue model:
  - ▶ Only the queue length is used



- ▶ Model does not include information about the queue dynamics
- ▶ New model with dynamics information:
  - ▶ Waiting time for all cars
  - ▶ Theoretically **infinite** state space
  - ▶ Necessary to use queue **estimation**
  - ▶ Basic dynamic model is also used by  
(Homolová J., Nagy, I., 2005)

# New Queue Modeling

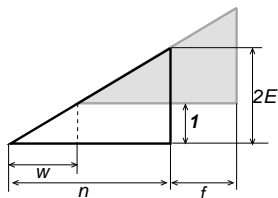
► Flow model:



- $E$  - mean value of waiting time
- $n$  - queue length

# New Queue Modeling

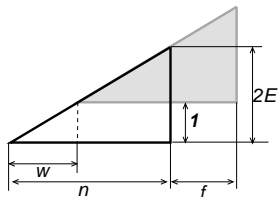
► Flow model:



- $E$  - mean value of waiting time
- $n$  - queue length
- $w$  - incoming traffic
- $f$  - outgoing flow

# New Queue Modeling

► Flow model:



- $E$  - mean value of waiting time
- $n$  - queue length
- $w$  - incoming traffic
- $f$  - outgoing flow

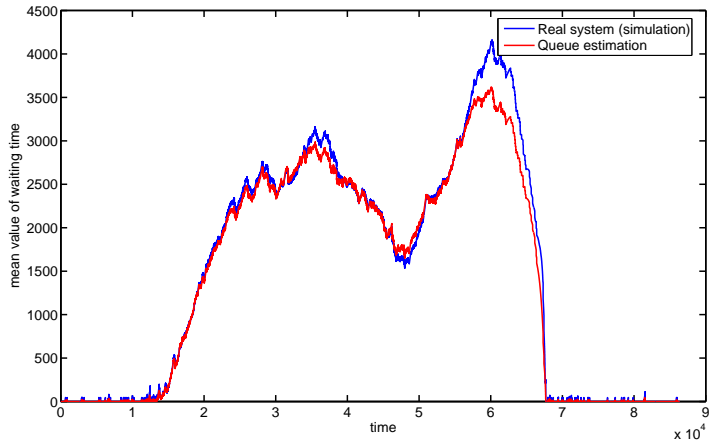
$$n(k+1) = n(k) + w(k) - f(k)$$

$$E(k+1) = \frac{\frac{E(k)(n(k)-f(k))^2}{n(k)} + n(k) - f(k) + \frac{w(k)}{2}}{n(k) - f(k) + w(k)}$$

► Inspiration from: (Henriksson, D., Abdelzaher, T., Lu, Y., 2004)



# New Queue Model Verification





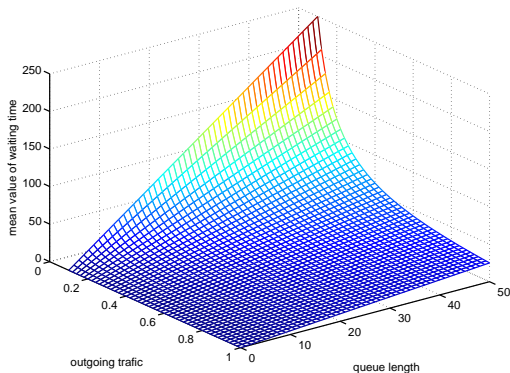


# Equilibrium

Equilibrium condition:

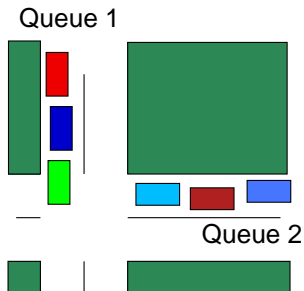
1.  $f^\circ = w^\circ$

2.  $E^\circ = 1/2 \frac{(2n^\circ - f^\circ)n^\circ}{2f^\circ n^\circ - f^\circ{}^2}$



# Intersection Model

- ▶ Crossroad model with two queues
- ▶ Parallel interconnection of queues
- ▶ Multiple input system
- ▶ Prepared for hybrid controller with safety time

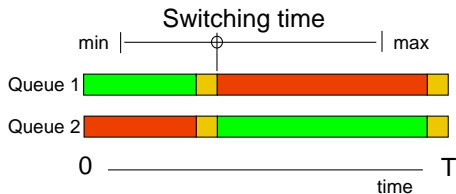


# Anti-collision Queue Control

- ▶ One period on the crossroad:

- ▶ Safe time  
(orange box) = 0

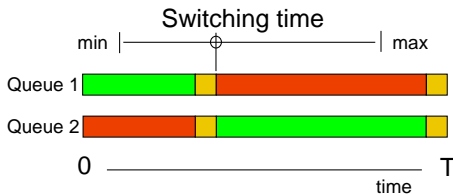
- ▶ Min = 0; Max = T



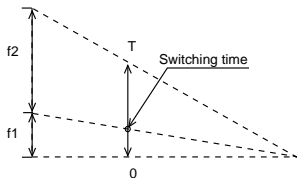
# Anti-collision Queue Control

► One period on the crossroad:

- Safe time (orange box) = 0
- Min = 0; Max = T



► Switching time computing:



- f1 - required outgoing flow for 1. queue
- f2 - required outgoing flow for 2. queue



# State Space and Linearization

- ▶ State vector:  $\mathbf{x} = (n_1, E_1, n_2, E_2)'$
- ▶ Input vector:  $\mathbf{u} = (u_1, u_2)'$
- ▶ Input disturbance vector:  $\mathbf{w} = (w_1, w_1)'$
- ▶ State variables:

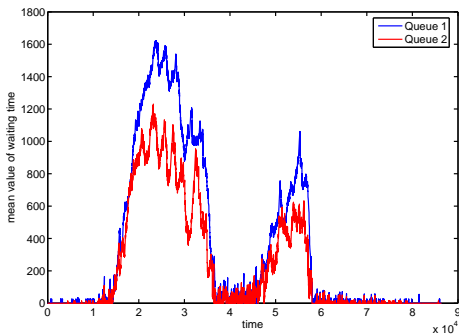
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.050 & 0.997 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0.020 & 0.998 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -1 & 0 \\ -17 & 0 \\ 0 & -1 \\ 0 & -11 \end{bmatrix} \quad \mathbf{B}_w = \begin{bmatrix} 1 & 0 \\ -17 & 0 \\ 0 & 1 \\ 0 & -11 \end{bmatrix}$$

- ▶ Linearization at point:
  - ▶  $f_1^{\circ}(w_1^{\circ}) = 0.03$ ;  $f_2^{\circ}(w_1^{\circ}) = 0.045$
  - ▶  $n_1^{\circ} = 20$ ;  $n_2^{\circ} = 50$ ;
  - ▶  $E_1^{\circ} = 333$ ;  $E_2^{\circ} = 555$



# LQ Controller

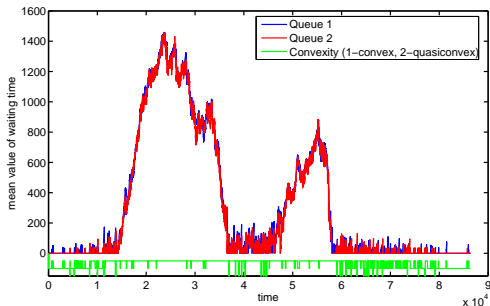
- ▶ Minimization function  $J = (E_1 - E_2)^2$
- ▶ Matrix  $\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$
- ▶ Control law  $\mathbf{u} = \kappa(\mathbf{x})$  is received as solution of the Riccati equation:  $\kappa = \begin{bmatrix} -0.001 & -0.020 & 0.000 & -0.012 \\ 0.001 & 0.016 & -0.001 & -0.062 \end{bmatrix}$





# NMPC Controller

- ▶ Minimization function  $J = (E_1 - E_2)^2$
- ▶ Predictive and control horizon 90 s

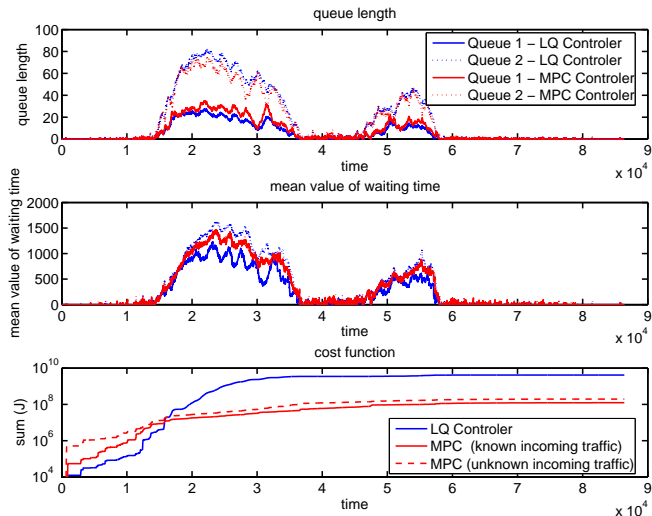


- ▶ Notices:

- ▶ Unknown incoming traffic  $\sum J = 19,6 \cdot 10^7$
- ▶ Known incoming traffic  $\sum J = 12,3 \cdot 10^7$
- ▶ Minimization problem (convex function?)



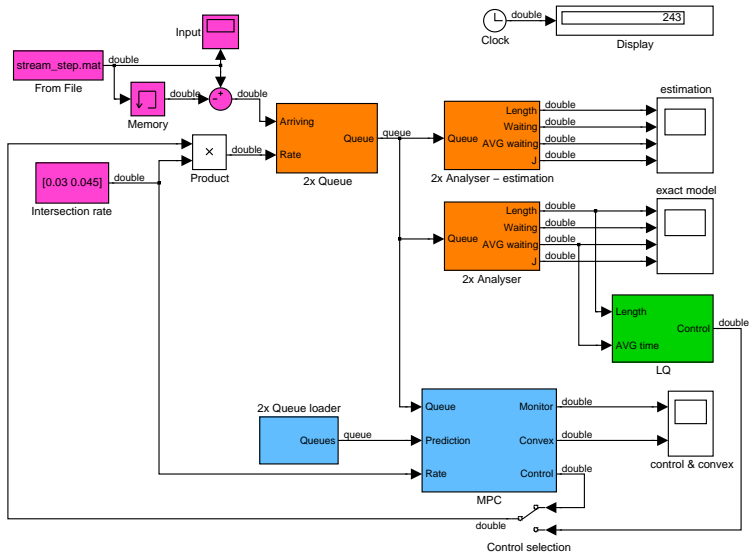
# MPC versus LQ Controller







# Simulation Model





## Future Work

- ▶ Verification by Petri-nets
- ▶ Algorithm for minimum cost function search
- ▶ Stability for NMPC controller  
*(Magni, L., Nicolao, G., Scattolini, R., Allgöwer, F., 2003)*
- ▶ Multiple intersections connection *(Lei, J., Ozguner, U., 2001)*



# Thank You ...

► Questions?